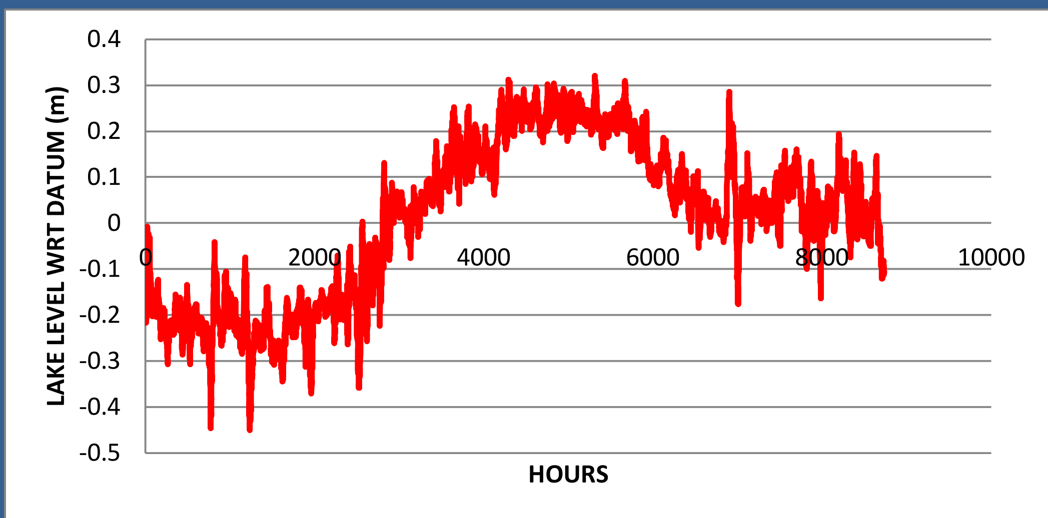
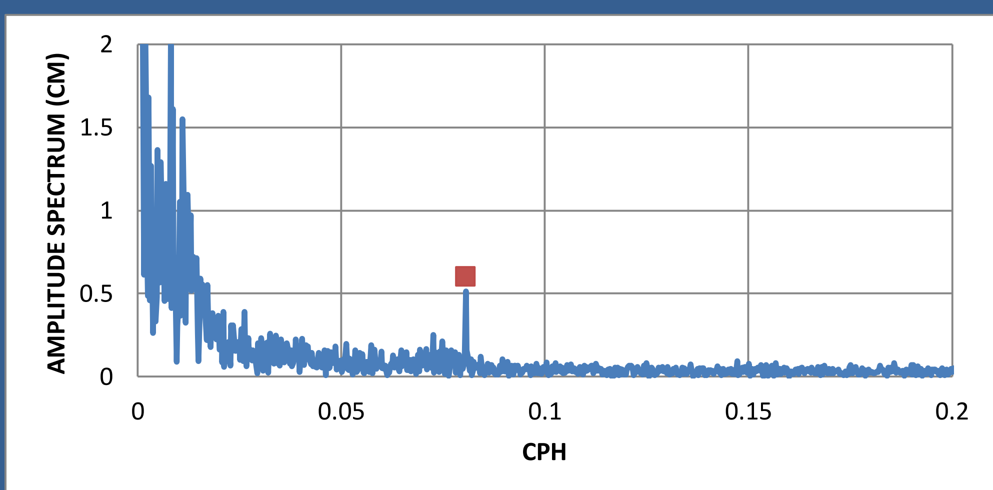
The Tobermory Tides

John Greenhouse, Tobermory, Ontario, Canada





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Biography

Dr. John Greenhouse is a geophysicist and board member of Sources of Knowledge. This paper reflects his interest in the dynamic mood of Lake Huron, which is clearly evident from his residence in Tobermory.

Introduction.

I became interested in this topic a few months ago when someone asked if there would be observable Lake Huron tides lapping up on the shores our home town of Tobermory, Ontario.. I had no idea. Most of us know that lake water levels change seasonally and over the long term, and that short term changes associated with storms and seiches can be observed. We would probably agree that bodies of water as large as the Great Lakes must have <u>some</u> tidal influence, but since we never actually see tides in our daily interactions with the lake we would assume that they must be very small. The point of this article is to show that, yes, there are measurable tides in Tobermory and indeed at most Lake Huron locations, that they are indeed small (on average only 5 to perhaps 40 millimetres in size), and that they are best seen through a different view of the water level data obtained via the Fourier transform. The treatment is not mathematically rigorous, but it does try to give some idea of the methodology, and to point out that it can be undertaken with software tools most of us already have.

A warning, however; physical limnology is a complex science that can be simplified only up to a point. If this article piques your interest, the references at the end provide a starting point to the broader subject.

Some tidal facts.

First a few words about tides. They are caused by the gravitational attraction of the Moon and Sun, with the lunar contribution being much the larger. Earth's rotation beneath the Sun (the S1 period) and the beneath the Moon every 24.84 hours (the M1 period) are the key factors in the varying tidal forces. These tidal forces are highest not just when these bodies are overhead, but also when they are directly underfoot (on the far side of the Earth) so the dominant tidal repeat time or "periodicity" is the lunar diurnal (12.42 hours,M2) with a much smaller contribution from the solar diurnal (12 hours, S2). The tidal range, the height difference between successive high and low tides, is determined by whether the Moon and Sun are pulling in line (the large Spring tides) or at right angles (the smaller Neap tides). Spring tides and Neap tides each occur roughly every 14 days. For a simple graphic of this please see the Tide Animation box or (for more detail) the Wikipedia in the Reference section at the end of this article.

Since we know that any tide observed in Tobermory will be very small to begin with, our main goal will be to examine the water level records for variations with a periodicity of 12.42 hours. We should also keep in mind the possibility of a much smaller S2 periodicity. But please note that this is a huge simplification; tide tables for a seaport may require involving as many as 62 tidal periodicities, taking proper account of the elliptical orbits of the Earth and Moon, the precession of these orbits, local bathymetry, and various other considerations[1].

Finally, it is more convenient, at least for what we do, to talk of M2 and S2 in terms of their frequency which is the inverse of their periodicity. The M2 frequency is therefore 1 cycle per 12.42 hours, or 0.0805 cycles per hour (CPH), and the S2 frequency is 1/12 or 0.0833CPH.

Previous work on Lake Huron tides.

Nothing, as they say, is really new under the Sun (or Moon in this case). The earliest account of tidal studies on Lake Huron I could find was written in 1929 by a cottager in the Little Current area, one R.K Young, who built a crude water level recorder and, over the course of 2 weeks, produced the record of Figure 1. There is a definite periodicity of peaks and troughs, repeating roughly every 12

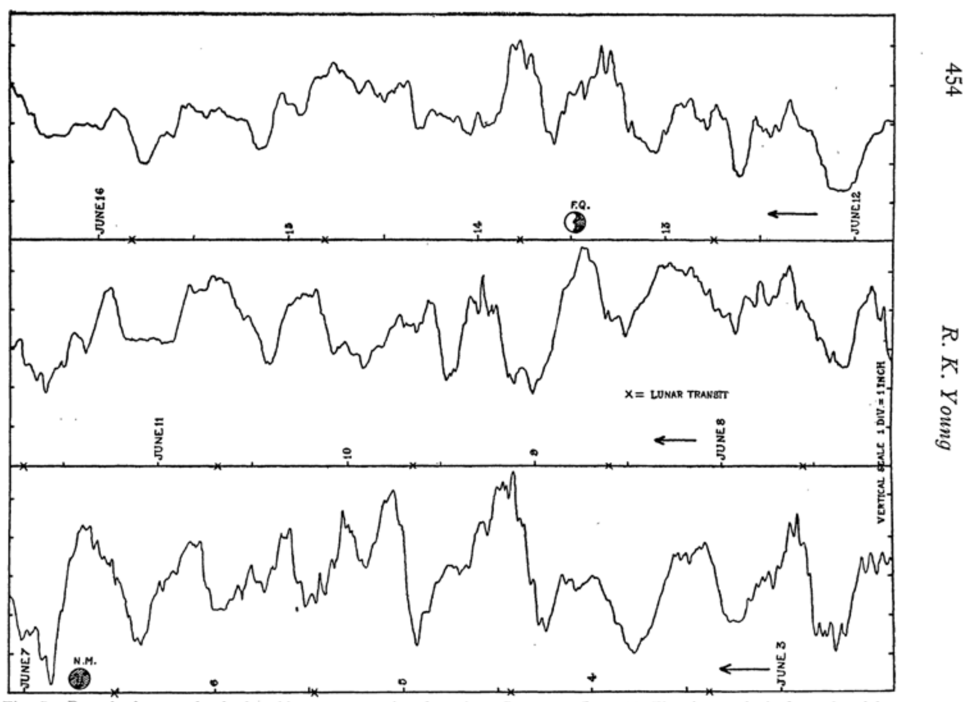

Fig. 7.—Record of water levels for thirteen consecutive days from June 3 to June 16. The time scale is from the right toward the left, as indicated by the arrows. The most prominent variation is the semi-diurnal tide. The positions of New Moon and First Quarter are marked and it will be noted that the daily tides are greater near New Moon. The time of lunar transit across the upper meridian is marked x.

Figure 1. A tidal record obtained at Little Current on a home-made recorder by cottager R.K. Young in 1929. The record covers 13 days, from bottom right to top left. The vertical scale marks are 1 inch (2.54 cm) apart. The horizontal scale marks are 12 hours apart.

hours with swings ranging from 2 cm to about 7 cm. Young identifies these peaks and troughs as tides. Young also mentions the work of W.J. Loudon in 1905, also published in the Transaction of the Royal Society of Canada but which I could not access, who measured tidal ranges of about 5 cm in

[1] Amongst these other periodicities are "harmonics" of the basic M1 and S1 periodicities because, like notes on a piano, these tidal frequencies are not pure sinusoids and contain energy at sub-multiples of the M1 and S1

Go Home Bay, a long narrow channel on the east shore of Georgian Bay. Another early study by W.D. Forrester (1961) also measured lunar tidal contributions to water levels at Little Current between 1.2 and 2.7 cm. Examples of more modern studies (Mortimer and Fee, 1976; Mortimer, 2006; Trebitz, 2006) are also listed in the References section.

Water level records.

Thanks to the web site of Fisheries and Oceans Canada[2], we do not have to build water level recorders for ourselves. Figure 2 shows examples from that web site, in this case the first 400 hours of 2011 in Tobermory, Killarney and Little Current. Measurements are made once per hour with an

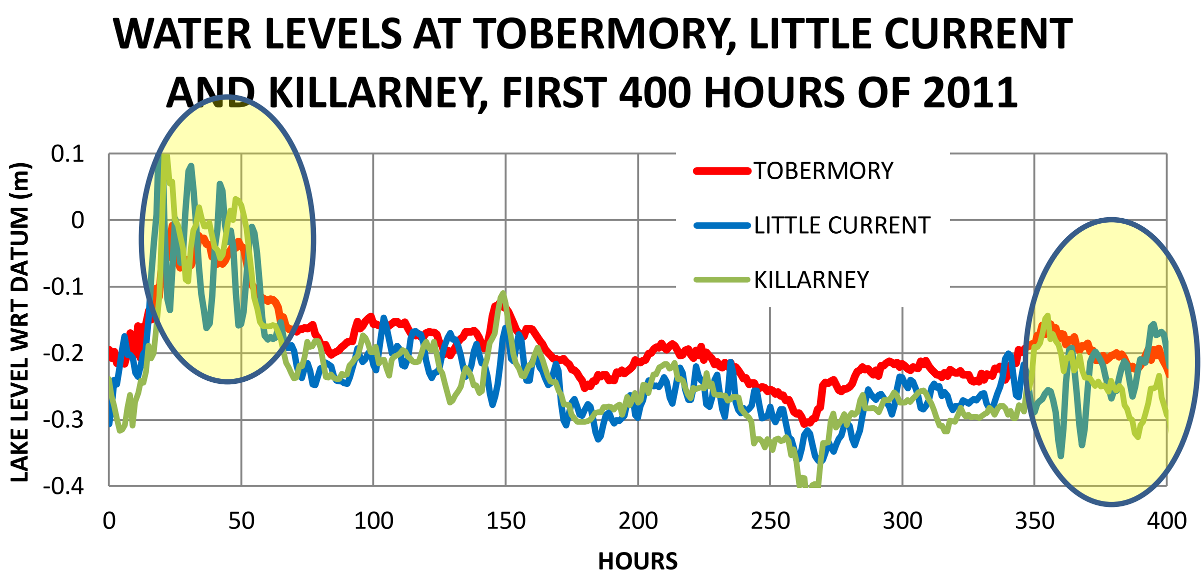

Figure 2. Water levels at Tobermory, Little Current and Killarney for the first 400 hours of 2011. Levels are measured relative to the Lake Huron datum. Highlighting ovals show periods of Spring tides.

accuracy of 1 millimetre. The major fluctuations in water level over these 16 days are due to winds and storms, winds which pile water up at one end of the lake and changes in barometric pressure which raise and lower the lake surface. Superimposed on these longer term changes, however, are prominent short term oscillations with a distinctly diurnal (~12 hour) period in the Little Current and, though more subdued, in the Killarney records. These are most obvious within the two high-lighted Spring tide periods on the left and right hand sides of the record. It seems reasonable to connect this ~12 hour oscillation with lunar tides, and to hypothesize (as Young and Forrester do) that they are amplified by the narrow channels at Little Current and Killarney through which tidal currents must pass.

But what about the Tobermory record, the red signal in Figure 2? Diurnal fluctuations might be

[2] http://www.waterlevels.gc.ca/C&A/gs_huron_e.html

identified in the left highlighted area between 25 and 50 hours, but this conclusion would not be backed up elsewhere in the record. Nevertheless, the Tobermory tides are there, and when the data are viewed a bit differently they can be seen quite easily. This is thanks in large part to the work of the great French mathematician and physicist, Joseph Fourier.

Figure 4. Joseph Fourier, 1768-1830 (image from Wikipedia).

Joseph Fourier and his Transform.

In the living room my grandson is playing with a "transformer" toy. This one starts out as a perfect little sphere and then, through a manipulation that only he can do, it transforms into a dune buggy. These two states bear no resemblance to each other, yet they contain exactly the same parts. The spherical properties of these parts are well displayed in the first state; the dune-buggy-ish properties in the second. Two views, same parts, with a means of transforming from one to the other. The Fourier transform described next is basically a mathematical version of that toy, a means of looking at data in two completely different ways.

Joseph Fourier was born in 1768 to a tailor and his wife, and was orphaned 8 years later. Despite these humble beginnings he rose to be Permanent Secretary of the French Academy of Science. Along the way he made major contribution to the decoding of the Rosetta Stone, to the theory of heat transfer, and to mathematics in general. One outcome of his work on heat was to recognize that our planet is warmer than it should be based on its distance from the Sun, and to recognize the contribution of what we now call the "greenhouse effect".

Fourier is probably best remembered today, however, for a mathematical procedure known as Fourier Analysis. It takes a set of data like our water level record and displays it in a quite different way, while providing the means (the "transform") of going between the two descriptions. This technique is mathematically quite complex but the concept is not really difficult and it can be undertaken with readily available software (in my case Microsoft EXCEL) to identify the tides at Tobermory.

Churchill, Manitoba.

A good place to demonstrate Fourier's approach is with a water level record from an ocean site, where the tides are obvious. Here I have chosen Churchill, Manitoba, on Hudsons Bay. Figure 5 shows water levels recorded once an hour for the first 1000 hours (roughly 42 days) of 2011. Each reading is an average over that one hour period, and is again recorded with a resolution of one millimetre. The record is dominated by very obvious M2 tidal oscillations – the ones with roughly 8 cycles every 100 hours. The tidal range, the difference between high and low tidal levels, is modulated by the fortnightly Spring/Neap tide cycle, varying from about 4 metres at Spring to 1.5 metres at Neap.

The genius of Fourier was to provide another method of portraying the information contained in

Figure 5, a method much better suited to a quest for repetitive fluctuations like tides. He showed that records such as this can always be reproduced by adding together a number of sine waves. The amplitude and frequencies of these sine waves together constitute what is known as the Fourier spectrum, and the means of performing this transformation is the Fourier Transform (FT).

This concept is in fact quite familiar to us. Most people know that white light contains a broad

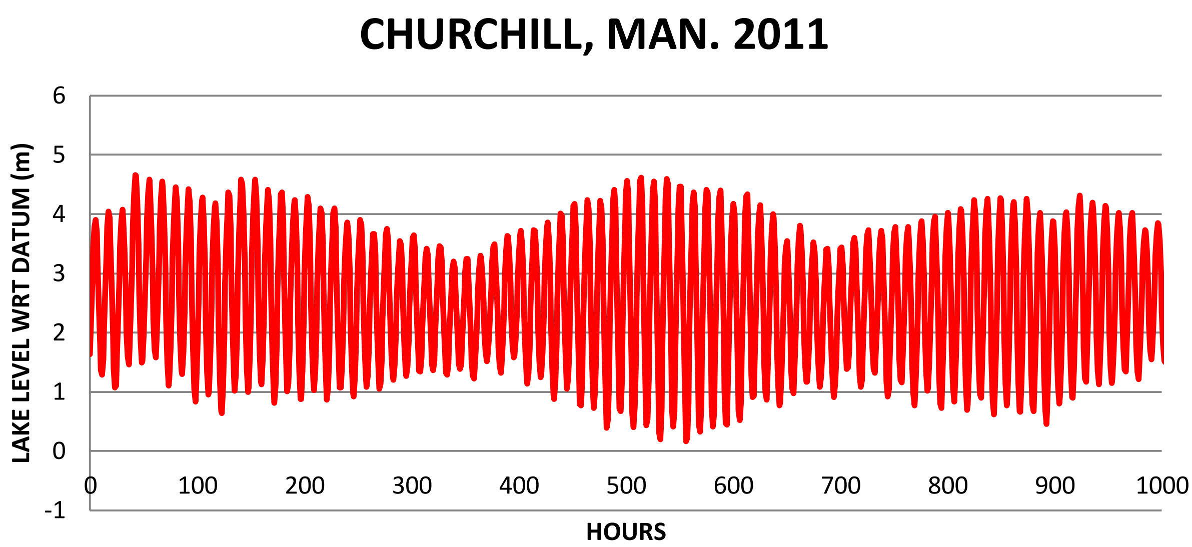

Figure 5. The record of sea water level in Churchill, Manitoba, for the first 1000 hours of 2011. Lake level is shown on the vertical axis as metres above a fixed datum. Time is plotted horizontally in hours. The record is comprised of one reading made every hour. Short period oscillations are the diurnal (M2) tides. The longer term variation in tidal range is caused by the spring/neap cycle

spectrum of colours, and each colour is associated with a frequency of its light. Digital cameras will in most cases display the spectrum of colours in a photograph, and allow us to diminish or enhance their contribution to the picture. Our stereos and iTunes software allow us to alter and balance the frequencies within the spectrum that makes up the sound we hear by adjusting their amplitudes. In each case we implicitly recognize that the sound or picture or colour can be synthesized by a summation of oscillations of varying frequencies. Water level records such as those depicted in Figure 2 or Figure 5 can also be synthesized by adding together a bunch of sine waves, each with an amplitude and a frequency. These sine waves collectively form the spectrum of the water level record. The two depictions of the record, the series of numbers in time and the spectrum of frequencies, look quite different but they contain exactly the same information.

The Fourier Transform for going back and forth between these two states would have been very cumbersome in the 18th and 19th centuries, but the so-called Fast Fourier Transform technique developed in the 1950s, plus the advent of digital computing, made this alternate view of data readily available If you are comfortable with the concept of a spectrum of frequencies, or just don't want more detail, skip the next section but look at the example in Figure 5 and Figure 6. For others, a few facts of the transform should make the concept clearer.

Some basic mechanics and an example of the Fourier transform.

The mathematically inclined will quickly realize that the treatment below is far from rigorous, so again please bear with my simplification. Starting with a record of N readings at N equi-spaced hourly times, the Fourier transform allows it to be reproduced by adding together N/2 sine waves at equi-spaced frequencies f. Each sine wave had an amplitude A and a phase ω. The phase recognizes that each sine wave may need to be shifted a portion of a cycle to make the summation work. The independent variable, in our case the N readings of water level, is now replaced by an N/2 amplitudes and N/2 phases. Like the N hourly times of the original record, the N/2 frequencies f are prescribed, in this case to be at intervals of 1/N cycles per hour (0), going from 0 to 0.5 CPH. For example, the 1000 data points of Figure 1 can be replaced by 500 sine waves, whose frequencies are 0, 0.001CPH, 0.002CPH, etc., all the way to 0.5CPH.

The more data points you have the more sine waves will be required for the summation, and the closer together their frequencies will be. We are interested in the amplitude of a very specific frequency, 1/12.42 or 0.0805 CPH, on the assumption that it represents the M2 tidal contribution to the water level record. To avoid contamination by energy at nearby frequencies, we want to achieve as high a resolution of that frequency as possible. This requires as large a data set N, or as long a

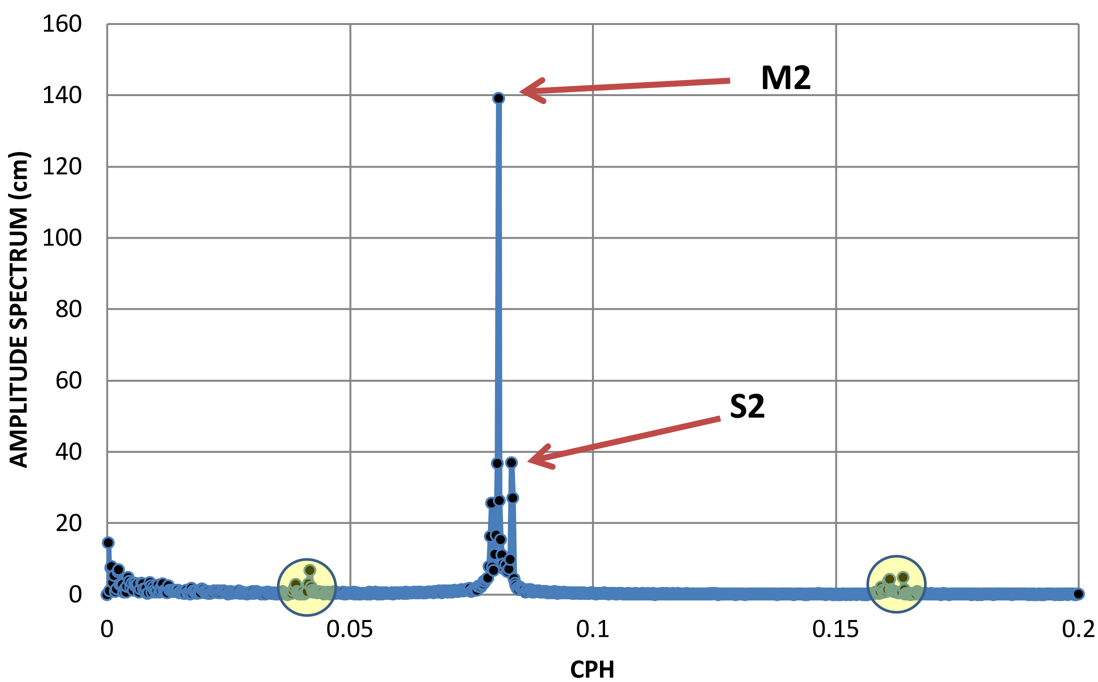

Figure 6 The Fourier amplitude spectrum of the water level record of Figure 5, limited to frequencies below 0.2CPH. Vertical scale is now in centimetres. The M2 and S2 tidal signals are identified. The M1/S1 and M4/S4 signals are tentatively identified by the small circles.

record, as possible.

Look now at Figure 6. In that figure the longest possible data set that my software can handle, 4096 hours or roughly the first 171 days of 2011 at Churchill, has been Fourier transformed, resulting in a Fourier amplitude spectrum. The frequencies of the 2048 constituting sine waves (that when added together will produce the original record of Figure 1) is plotted along the horizontal axis, while the amplitude of each of these sine waves, now in centimetres rather than metres, is shown on the vertical axis. The 2048 individual points on the graph are joined by a smoothed line. Strictly speaking this should be accompanied by a phase spectrum, consisting of the 2048 ω values as a function of frequency, but for our purposes this can be ignored.

The term "energy" has been used above to refer to the contribution of each frequency sinusoid to the overall spectrum. This is a bit loose with the physics but it helps the discussion. On Figure 6 it is clear that most of the energy is contained at the point with amplitude 140 cm right at the M2 tidal frequency of 0.0805 marked by the arrow. The amplitude of this sinusoid, 140 cm, is a measure of its size with respect to its baseline, so the range (from peak to trough) would be 280 cm.

So far the Fourier view of the Churchill data tells us what we already knew; the sea level record there is dominated by lunar diurnal tides which repeat every 12.42 hours. But there is also more subtle information here. Although the M2 tide is largest, as expected, there is a smaller but significant S2 solar peak in Figure 5, with a period of exactly 12 hours and frequency of 1/12 = 0.0833CPH. Churchill's tides do have a solar component, though its presence would never have been evident from the original record of Figure 5. If the M2 and S2 amplitudes are added (180 cm, or a 3.6 metre range) we get a better fit to the average tidal range of Figure 5. (From this we can assume that the M2 spectrum amplitude may always underestimate the actual tidal range.) Furthermore, the small double peaks in the vicinity of 0.04 and 0.16 CPH are the S1/M1 and M4/S4 harmonics of the main tidal signal. On a vertically expanded scale these can be clearly distinguished.

To summarize, all the information in the record of water level with time (the "time series" of Figure 5) can also be completely represented by the amplitudes and frequencies of a set of sine waves (the "Fourier spectrum" of Figure 6). Each representation has advantages and disadvantages. For example, If you want to know what the high tide level in Churchill was on February 26th, 2011, the Fourier spectrum of Figure 6 does not help at all. From the time series of Figure 4, however, it would be easily determined.   But if you want to find an average contribution of solar tides or their harmonics at Churchill, the Fourier spectrum is really your only option. Most importantly, the Fourier transform allows us to go back and forth between these two representations of sea level depending on how we want to examine it.

Tobermory.

From this we can turn again to the water level record at Tobermory. We know that water levels on Lake Huron are going to be dominated by seasonal and barometric (storm and wind) variations, and that tides will be very small in comparison. The hope is that the Fourier representation of the data can tease the tidal information out of this large background and we can be optimistic based on the fact that the tides, unlike the storms, have that very predictable frequency of occurrence.

Figure 7 shows the entire water level record at Tobermory for 2011. Once again the readings are made every hour, and the vertical axis shows lake level in metres relative to lake datum of 176 metres above sea level. We see the dominant seasonal variation, low water in winter, high water in summer, and a lot of sharp spikes due to storms which cause the lake to slosh back and forth for a few hours or days as seiches. The total range of lake levels is about 70 cm. Even on an expanded

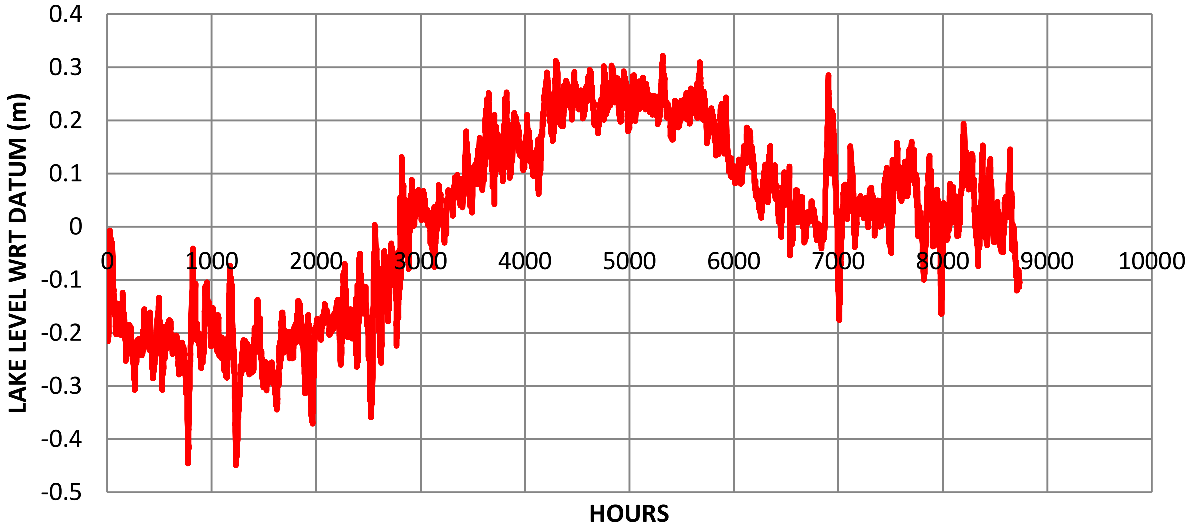

Figure 7. The water level recorded at Tobermory for 2011. Readings are made hourly, and the vertical scale is metres with respect to Lake Huron datum.

scale (e.g.Figure 1) there is no unambiguous sign of a tidal signal.

Figure 8 shows the Fourier series representation of the first 4096 hours of that record (the maximum that EXCEL will accept at one time). Once again the amplitude of the individual sine waves is plotted in centimetres on the vertical axis, and the frequency of the sign waves along the horizontal axis. We can see that there is not a lot of energy, and the highs and lows are random and poorly defined, at frequencies above 0.1CPH. The remainder of the record extending to 0.5 CPH is similar and the spectrum in Figure 8 has accordingly been truncated at 0.2CPH to give better resolution at the frequency range of interest. For the same reasons the vertical scale has been limited to 2 cm., even though storm and seasonal variations are much larger than that. The symbols at each of the 2048 constituent frequencies are also omitted, leaving just the line that joins them.

Remarkably, because it has an amplitude less than 1cm, the M2 tidal signal is clearly resolved in Figure 8 by the prominent peak at 0.0805CPH marked with the red square. An analysis of the last 4096 hours of 2011 shows exactly the same peak, so it is not an artifact of the computation. As with Churchill, we can expect the doubled M2 peak (1 cm) in Figure 8 to slightly underestimate the average tidal range at Tobermory[3]. As expected, most of the energy in the spectrum is below

[3] The highlighted area of the Tobermory water level record (red line) near 50 hours in Figure 2 shows fluctuations of about

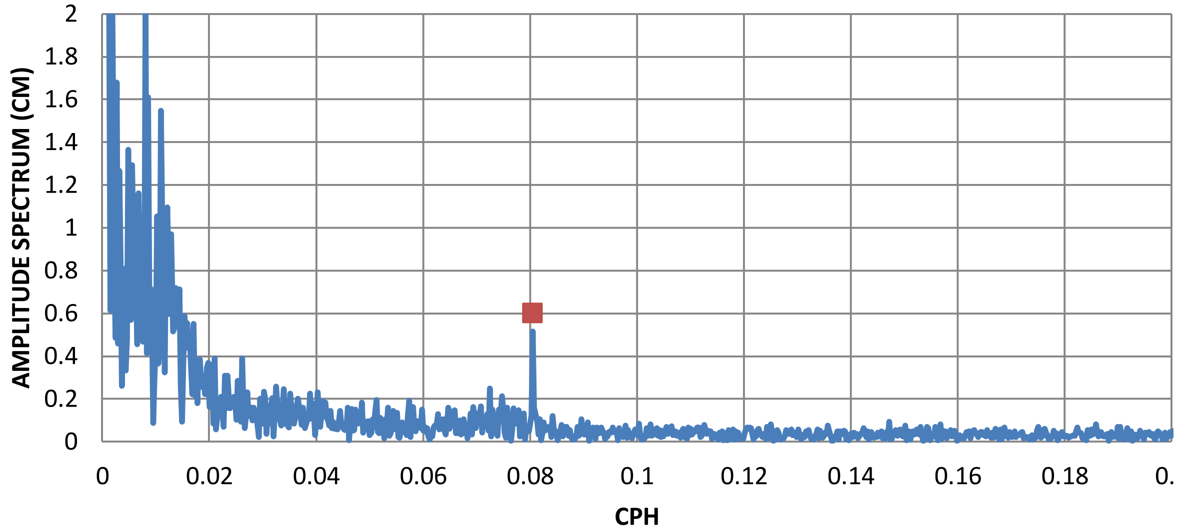

Figure 8. The Fourier spectrum of the Tobermory water level record for the first half of 2011. Note the prominent M2 tidal peak at 0.0805 CPH, marked by the red square. The upper limits of the vertical and horizontal axes have been truncated at 2cm and 0. 2CPH respectively to better display the frequencies of interest.

0.02CPH (variations greater than 50 hours or roughly 2 days), mainly representing barometric changes. The seasonal range of about 60 cm in Figure 1 would be found at the very lowest frequencies, whose amplitudes have been truncated in Figure 8. No solar S2 peak is discernible; at Churchill S2 amplitude was about 25% that of M2 so that it would reasonably have been lost in the background at Tobermory.

Other Lake Huron water level recording stations.

If there are indeed measurable tides at Tobermory then they should also show up on the water level records of other Lake Huron stations. Figure 9 compares the spectra of the water levels at Tobermory, Killarney, Goderich, Little Current, Collingwood and (on Lake Ontario) Kingston for the first 4096 hours of 2011.

In all cases there is evidence of the M2 tide, highlighted by the vertical red-shaded column. Kingston on Lake Ontario also shows a small M2 tide. Tidal ranges, doubled peak amplitudes, vary from 1 to 4 cm. Note that the peak at Little Current is considerably larger than the rest, as suggested earlier the result of amplification of the tide by the narrow channel through which it must pass. At Killarney, with the second largest M2 peak and also situated in a narrow channel, the same hypothesis might hold.

The red and yellow squares on the upper plot identify the M1,M2 (red) and S1,S2 (yellow) tidal frequencies. At Killarney, where tidal responses seem to be enhanced, there may also evidence of the solar tide S2. At several of the stations (though not Tobermory) there appears to be energy at the S1 frequency, one cycle per 24 hours. These small peaks are quite distinct from the lunar day signal

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3-4 cm. These <u>may</u> be an indication of spring tide amplitude.

of 24.84 hours, and perhaps represent a diurnal contraction/expansion cycle due to solar heating and cooling.

Finally, note the slight bump in energy, at Goderich and Little Current, in the vicinity of 0.15 cycles per hour, corresponding to repetitive rise and fall of water level roughly every 7 hour period. These bumps are not much above the background level and it would be unwise to make too much of them, but they are within the range of frequencies predicted for the Lake Huron and Georgian Bay seiches.

In summary.

That there are tides on the Great Lakes has been recognized for at least a century. No new ground has been broken with this article; its contribution is simply to point out that, when water level records are viewed in the Fourier domain, tides can be recognized and measured at locations where they are otherwise hard to observe. I have applied this analysis to most water level recording stations located around Lake Huron, Canadian and American, and they all show that distinctive M2 tidal line with amplitudes of 0.5 to 2.5 centimetres. With readily available software we can all analyze the digital water level records for this interesting force of nature[4].

But is it useful to do so?! The 1960 study by Forrester referred to above was undertaken to see if tides could be used to predict the variable (in speed and direction) current flows through the North Channel at Little Current. He concluded that they could not, that barometric swings, winds and their associated seiches were the dominant considerations in determining currents. The overall drop in lake levels, and the damage that can be caused by seiches, are clearly more important topics for study than the comparatively minuscule influence of these tides.

Still, some things are worth knowing about even if they are of no practical use, and this may be one of them. If nothing else, we have an answer for the next time someone asks if there are Tobermory tides.

Acknowledgements.

Thanks to Scott Parker and my family for critical readings of this article. Any remaining errors are mine.

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[4] Anyone interested in the computational details of Fourier analyzing tidal records with a spreadsheet can contact me: jpgreenh@amtelecom.net

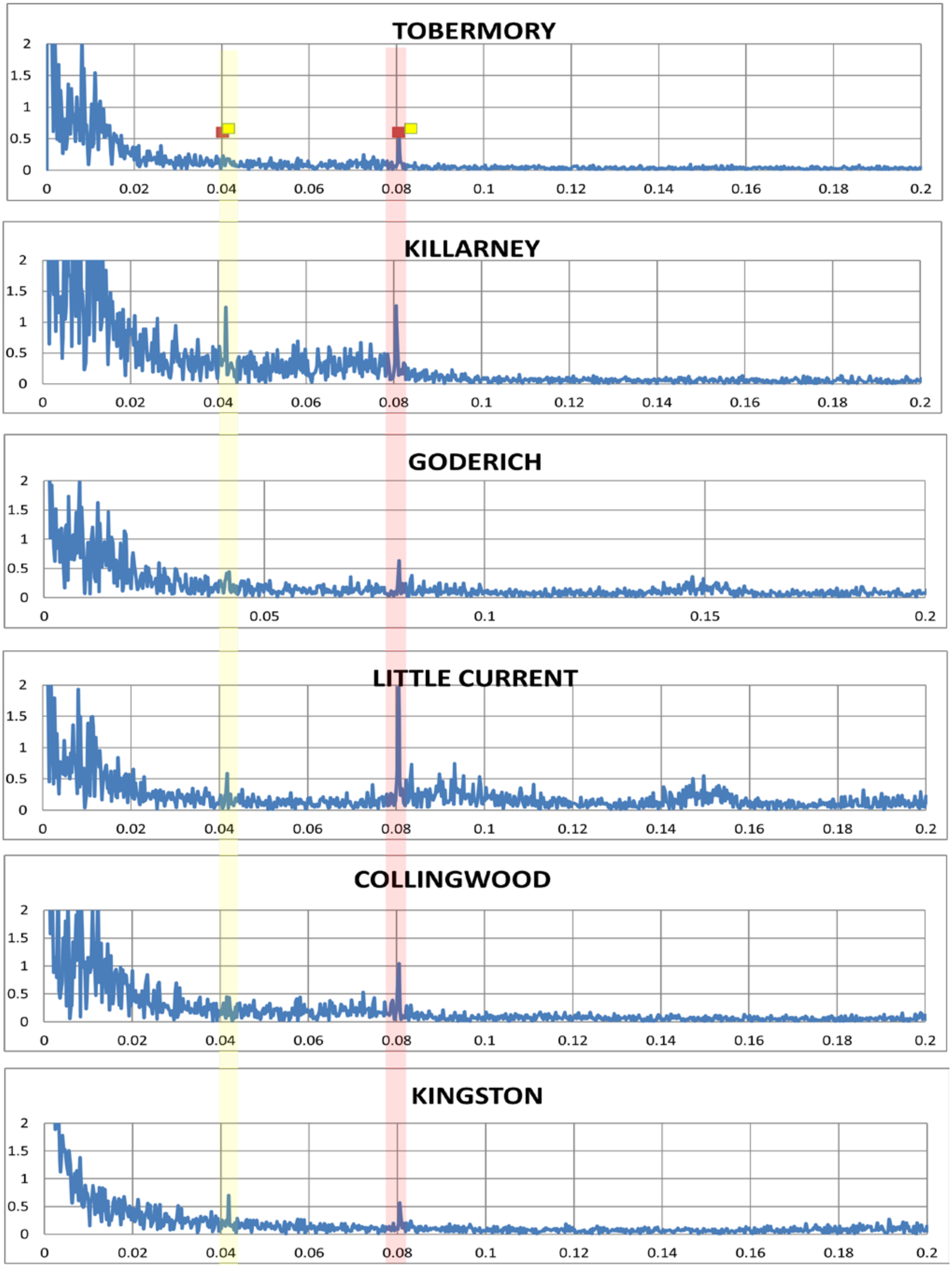

Figure 9. Comparison of water level spectra for five stations on Lake Huron, and for Kingston on Lake Ontario, for the first 4096 hours of 2011.. The vertical scales on all plots are spectral amplitude in cm, and the horizontal scales are frequencies in cycles per hour. The red and yellow squares on the upper plot indicate the M1, M2 and S1,S2 frequencies respectively. The shaded columns highlight the M2(red) and S1(yellow) frequencies.

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TIDE ANIMATION

TIDES BY WIKIPEDIA